tained in these situations are

$$p(r) = ar \exp(-r^2/2\sigma^2) \quad \text{(two-dimensional situation)} \quad (8)$$

$$p(r) = ar^2 \exp(-r^2/2\sigma^2) \quad \text{(three-dimensional situation)} \quad (9)$$

where $\sigma = \sigma_1 = \sigma_2 =$ (in the three-dimensional situation) σ_3. When the variances are not equal the process must be done numerically. This has been done for a wide range of points in the two-dimensional situation by Baur.[4]

Another approach is presented here. It was developed to fulfill the following objectives: 1) the method should have a relative error less than 1%; 2) it should work for both the two-dimensional and three-dimensional situations; 3) it should form the basis of a fast, efficient subroutine for digital computers; 4) it should provide the r probability distribution rather than just the points thereof.

Method

The method is based on the fact that certain moments of the distribution [Eq. (7)] may be obtained analytically. The r moments may be defined as follows:

$$\mu_k(r) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r^k f(\delta_1,\delta_2,\delta_3)d(\delta_1)d(\delta_2)d(\delta_3) \quad (10)$$

where $r^2 = (\delta_1)^2 + (\delta_2)^2 + (\delta_3)^2$

Integration by parts yields the even moments of the distribution, and, in particular, the following:

$$\mu_2 = (\sigma_1^2 + \sigma_2^2 + \sigma_3^2)\mu_0 \quad (11)$$

$$\mu_4 = [3(\sigma_1^4 + \sigma_2^4 + \sigma_3^4) + 2(\sigma_1^2\sigma_2^2 + \sigma_1^2\sigma_3^2 + \sigma_2^2\sigma_3^2)]\mu_0 \quad (12)$$

where μ_0 is arbitrary and has already been taken to be unity.

Let us assume that the distribution as a function of miss distance (r) always takes the form of Eqs. (8) and (9), namely

$$p(r) = ar^b \exp(-cr^2) \quad (13)$$

Then one may determine the parameters a, b, and c so that the 0th second and fourth moments of this distribution are the same as those of the distribution of Eq. (7). Integration by parts will also give the even moments of this distribution [Eq. (13)].

$$\mu_2 = [(b + 1)/2c]\mu_0 \quad (14)$$

$$\mu_4 = [(b + 3)/2c]\mu_2 \quad (15)$$

where again μ_0 is taken as unity.

Then b and c are found as follows:

$$b = (3T - 1)/(1 - T) \quad (16)$$

$$c = (b + 1)/2\mu_2 \quad (17)$$

where $T = \mu_2^2/\mu_4$ and the numerical values of μ_2 and μ_4 are taken from Eqs. (11) and (12). The normalizing parameter a is found by evaluating the integral

$$I = \int_0^{\infty} r^b \exp(-cr^2)dr$$

which may be expanded as follows:

$$I = \frac{r^{b+1}}{b+1} - \frac{cr^{b+3}}{b+3} + \frac{1}{2}\left(\frac{c^2r^{b+5}}{b+5}\right) - \frac{1}{3!}\left(\frac{c^3r^{b+7}}{b+7}\right) + \ldots \quad (18)$$

A value of $r = 5/(2c)^{1/2}$ was found to be sufficiently close to infinity to evaluate the total integral.

Finally,

$$a = 1/I \quad (19)$$

Once the parameters a, b, and c are found, the expansion [Eq. (18)] may be used to find the "cumulative" probability.

Results

The distributions obtained by this method were found to have errors of less than 0.5%, based on the error in μ_1 that was computed numerically for a number of test cases. In the two-dimensional case the agreement with Baur is within 1%. Four examples of the distributions obtained by this method are given for a two-dimensional target (Fig. 1). These are obtained by setting one of the variances (σ_3^2) to zero in the foregoing equation. No figures appear for the three-dimensional case because covering an interesting range of values for the three variances requires too many figures for an article of this size.

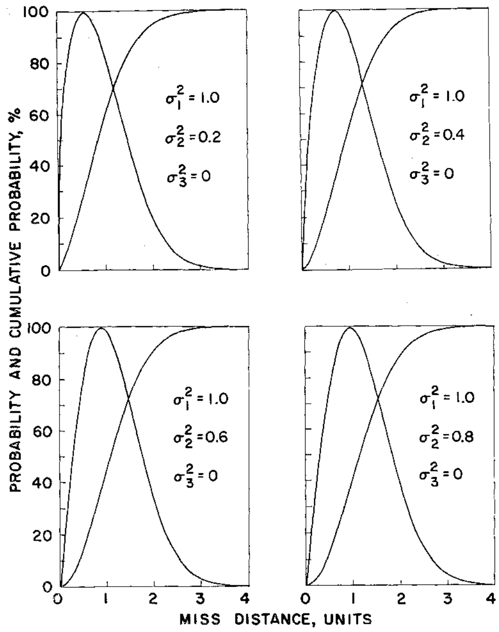

Fig. 1 Distributions obtained for a two-dimensional target.

All calculations described above have been coded in the FORTRAN language and are available from the author.

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Low Pressure Rocket Extinction

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Introduction

THERE has been considerable interest recently in designing solid rockets for low pressure operation because of the shell weight economy this would allow. Two primary requirements for this type of design are that there be no loss of combustion efficiency and that the rocket ignite

Received August 20, 1963. This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. NAS 7-100, sponsored by NASA.

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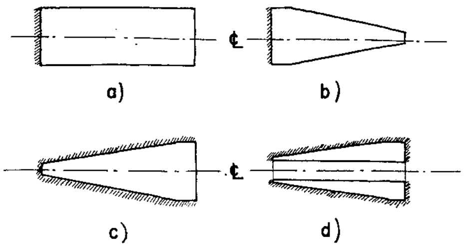

Fig. 1 Grain geometries used in first series of firings. All grains have cylindrical symmetry around the centerline. Shaded areas are inhibited. a) cylinder, b) cone, c) end burning cone, d) regressive tubular. Head end of rocket was to the left except for shape d), which was mounted with the head end to the right or left on alternate runs.

and burn in a completely reliable manner. Low pressure operation has always had its difficulties, however, and there is extensive literature concerned with observations of "chuffing" and other "ignition problems" at low pressure.[1, 2]

Early work in this laboratory showed that a solid rocket's extinction pressure P_e, which was usually well above the propellants' lowest stable burning pressure (Crawford bomb), correlated well with the port to throat ratio H_i for internal burning tubular charges.[2] These data were taken at atmospheric pressure where it was found that the system would either burn stably after the ignition transient or chuff repeatedly until the equilibrium internal pressure was high enough to cause stable burning. A few firings with regressive cylindrical rounds showed that the grain could extinguish without the presence of an ignition transient and therefore that the phenomena was probably caused by an inherent low pressure instability in the steady burning rocket system.

This paper reports the results of some further regressive grain firings which indicate that for any particular propellant, low pressure extinction is, in fact, directly and simply related to the L^* (free chamber vol/throat area) of the chamber at the time of extinction.

Experimental Data

All the data reported here were taken by burning regressive grains to extinction in a 3 in. diam variable length rocket motor under vacuum conditions. The details of the experimental set-up are given in another report.[3] In all cases the propellant burned for at least 1 sec before extinction occurred. Two sets of experiments were performed. The first consisted of 39 firings using JPL 534 (a composite with about 2% aluminum) as the propellant. In this series the chamber

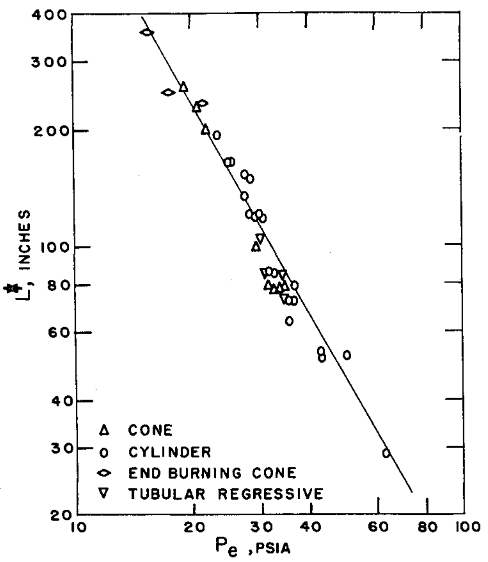

Fig. 2 Extinction pressure correlation for JPL 534. The slope of the line is −1.80.

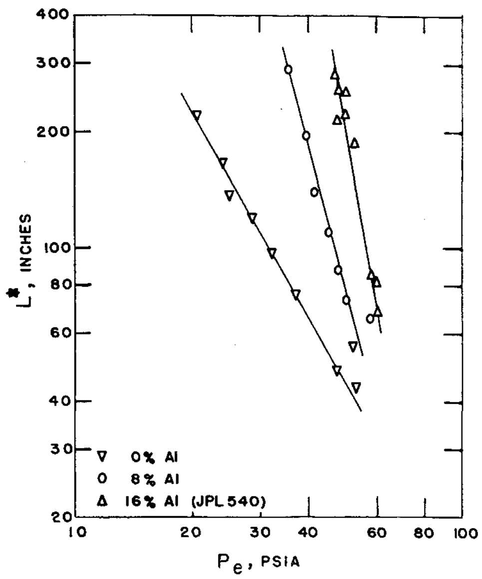

Fig. 3 Extinction pressure correlation for JPL 540 and modified JPL 540 propellants. JPL 540 contains 16% aluminum and the two modifications were made by reducing the aluminum content to 8% and 0%. The slope of the lines are 0%, −1.77; 8%, −3.97; 16%, −5.85.

length was varied from $12\frac{1}{2}$ in. to $4\frac{1}{2}$ in., and the grain geometry was changed to include the shapes shown in Fig. 1. Extinction pressure was found to vary from 64 to 16 psia and is plotted vs L^* in Fig. 2. In a few instances sufficient propellant remained after extinction to allow another firing with a smaller nozzle so that some of the points in Fig. 2 represent the same propellant charge extinguishing under two different nozzle conditions. The Crawford bomb extinction pressure for this propellant (in a nitrogen atmosphere) is 6 psia.

The second series of experiments were conducted to determine the effect of solid combustion products on the L^* correlation and to check that the correlation would be valid for a different propellant. In this series only right cylindrical grains (shape a, Fig. 1) were used. The data are plotted in Fig. 3. Notice that the presence of solid particles in the product gases markedly reduces the pressure sensitivity of the L^* correlation.

In about one-third of the firings, extinction was preceded by an increasing amplitude low frequency oscillation (5 to 40 cps). For JPL 534 the frequency of this oscillation correlated well with the L^* of the chamber. These spontaneous oscillations occur at a frequency well below the characteristic acoustic frequency of these chambers and, in fact, no phase shift was detected in the few firings that had pressure gages mounted at both ends of the rocket chamber.

Theory of the L^* Correlation

The low frequency oscillation sometimes observed before extinguishment is quite definitely a resonance phenomena related to the characteristic exhaust time of the chamber and is quite probably a manifestation of the instability that leads to extinguishment. The authors feel that extinguishment occurs in this dynamic flow system when the characteristic time of propellant burning-rate overshoot (for a slight pressure fluctuation) is of the same order of magnitude as the characteristic exhaust time of the rocket chamber. The burning-rate overshoot phenomena has been discussed theoretically in four recent papers[4–7] and its characteristic time is a function of the steady combustion pressure, increasing as the pressure decreases. On the other hand, the characteristic response time of the exhaust process to a slight fluctuation in chamber pressure is simply related to L^* and is independent of the pressure level in the rocket chamber. Thus, in our experiments, the operating pressure slowly decreased until the system became dynamically unstable and this instability led to extinguishment. It is interesting to note that this type of dynamic instability has been treated by Akiba and Tanno[8] and their linearized theory predicts that the slope of the L^* vs P_e correlation should be $(-2n)$ where n is the pressure exponent of the propellant. For JPL 534, $n = 0.86$ and therefore our data on this propellant agree quite well with their linearized theory.

At the present time there is no theoretical treatment for propellants containing solid products. However, one would expect these propellants to exhibit less pressure sensitivity because the presence of solids in the burning zone should lessen the effect of pressure level on the characteristic overshoot time. This was observed in the aluminized propellant (Fig. 3).

Conclusions

This preliminary work does not answer all the questions concerning low pressure extinction, but it does indicate that a few experimental vacuum firings using regressive grains can be used to determine the low pressure limit for any particular propellant formulation. This work also indicates that the theory of the phenomena needs further study, particularly for the case of propellants that yield a sizeable concentration of solid products when they burn.

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Simulation of Solid Propellant Exhaust Products with a Hybrid Rocket Motor

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SEVERAL major disadvantages have always been associated with the use of solid propellant rocket motors for certain types of test programs. These difficulties are summarized in the following paragraphs:

1) The cost of tooling is often prohibitive. Tooling costs are often encountered for each individual test, since test parameters cannot always be met with existing tooling. Propellant preparation and fabrication is expensive, due to the hazards involved. Remote handling must be provided, which results in high equipment costs and reduced efficiency.

2) There is a lack of test control. Once ignited, it is difficult to exert control over the solid propellant firing. In testing developmental rocket motor components, it is often desirable to test progressively (i.e., short firings of increasing duration). In such a program, it is not unlikely that failures will occur. In this event, the firing should be terminated immediately so that the nature of the failure may be determined before the unit is destroyed. Thus, it would be possible to repair and modify the unit and repeat the test. To accomplish this at present, short solid propellant firings must be scheduled, making it necessary to use a new motor each time. In the over-all picture, these short firings cost almost as much as longer ones. Progressive type testing, therefore, becomes prohibitive in cost and time.

3) Once ignited, the propellant is difficult to extinguish, and once it is extinguished, the grain cannot be reused. With regard to test termination, it is possible to install a blowout port or a similar device. However, once the propellant is ignited, it can be extinguished only through proper loss of pressure at a given rate. If a test is terminated, the entire propellant is lost along with the associated costs.

To avoid these difficulties inherent in solid propellant testing, attempts have been made to simulate the solid propellant exhaust. These have primarily consisted of adding dry powders to a hydrogen-oxygen rocket motor. Control of the flow rate of these powders, however, has proved to be difficult, and these simulation methods have not been completely successful.

In recognition of these problems, a hybrid rocket motor was conceived which would circumvent all the disadvantages just listed, while providing a reasonable simulation of the solid propellant exhaust products. Briefly, the solid propellant constituents, without oxidizer, are formed by conventional molding techniques. This composition constitutes the "grain." The oxidizer is then supplied externally through an injector. With removal of the oxidizer, such a grain can be molded safely in standard fabrication areas with minimum cost in tooling. A wide range of grain binder materials can be used, since the hybrid concept eliminates the usual stringent physical property requirements. Unbonded fuel grain segments, fabricated by a room temperature compaction molding process using temporary tooling, would be possible.

The simplest solid propellant formulation to simulate with the hybrid motor concept is an ammonium perchlorate-aluminum-binder system. The basic constituents of such a system may be represented chemically as,

$$NH_4ClO_4 + x(Al) + y(C_aH_b)$$

where C_aH_b is the organic binder.

It should be noted that by removing the oxygen from the ammonium perchlorate, it becomes ammonium chloride, a stable salt. Ammonium chloride, aluminum powder, and organic binder can be efficiently molded into a grain by using conventional techniques. The oxygen can then be supplied externally, resulting in the following chemical form:

$$2O_2 + NH_4Cl + x(Al) + y(C_xH_y)$$

Chemically, the only difference in the two chemical systems just depicted is the heat of formation of ammonium perchlorate, which is 3300 cal/mol. This results in a slight lowering of the combustion temperature in the simulated system. An estimate of this temperature difference may be obtained by assuming that this heat is removed after combustion, as follows:

$$3300 \frac{\text{cal}}{\text{mol}} \times \frac{\text{mol}}{117 \text{ g}} = 28.2 \frac{\text{cal}}{\text{g } NH_4ClO_4}$$

Assume the propellant composition to be simulated containg 60% NH_4ClO_4. For every gram of NH_4ClO_4, there are 1.67 s

Received August 28, 1963.

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